Estimation of Human Arm Angles Using Hand Pose Data and Upper Arm Radial Acceleration Measurements

Matjaž Mihelj, Marko Munih

University of Ljubljana Faculty of Electrical Engineering Tržaška c. 25, 1000 Ljubljana, Slovenia
E-mail: [matjaz.mihelj, marko.munih]@robo.fe.uni-lj.si

***Abstract*— The paper considers a novel technique for computation of the inverse kinematic model of the human arm, based on measurements of the hand position and orientation, and radial acceleration of the upper arm. The algorithm gives sufficiently good estimates of the human arm angles that can be later used in trajectory planning for rehabilitation robots, and evaluation of motion of patients with movement disorders.**

Ⅰ. INTRODUCTION

Upper-limb orthotic systems have been designed for restoring the upper limb functions of individuals with disabilities resulting from spinal cord injury (SCI), stroke, and muscle dystrophy. These systems employ either functional electrical stimulation (FES) or external power. In the past decade, a new kind of rehabilitation devices has emerged including the rehabilitation robots and haptic interfaces [2]. Kinematic analysis of limb movements can be used to evaluate motion of patients with movement disorders. Thus, it becomes an important issue in rehabilitation. In this regard, it is often relevant to measure the respective joint angles, together with the endpoint position of the arm. Moreover, when applying electrical stimulation to the upper-limb, the knowledge of the arm configuration is necessary to properly select the required stimulation pattern. Therefore, it is necessary to calculate the inverse kinematics of the limb. However, without the knowledge of the arm configuration, resolved from the redundancies, it is not possible to compute inverse kinematics for a given end point position. When a rehabilitation robot or haptic device is used, the position of the attachment point of the mechanism to the human arm is already known from the kinematic model of the device. Therefore, it becomes reasonable to use these existent data with minimal additional measurements to cope with redundant degrees of freedom of the human upper extremity. The aim of this work is the estimation of human arm angles using hand pose data and upper arm radial acceleration measurements.

Ⅱ. METHODS

The kinematic chain shown in Fig.1 consists of seven

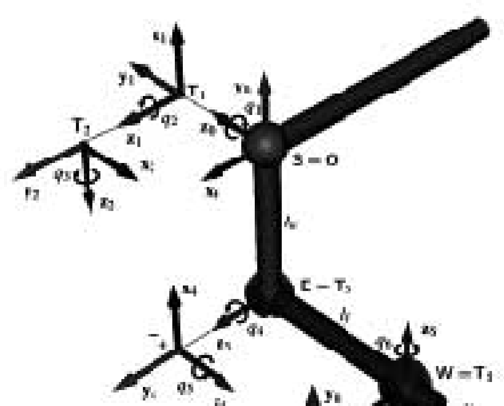

Fig. 1. Human arm kinematics

joint variables. It has one redundant degree of freedom. A simple physical interpretation of the redundant degree of freedom is based on the observation that if the wrist is held fixed, the elbow is still free to swivel about a circular arc whose normal vector is parallel to the straight line connecting the shoulder and the wrist (Fig. 2) [3]. The workspace of this mechanism was systematically analyzed by Korein [1]. Since there is a redundant degree of freedom, the exact inverse kinematics is not unique unless further measurements are considered in addition to the measurement of the hand position and orientation. The algorithm proposed in this paper uses an extra constraint to estimate the exact elbow position. Through the following presentation, it will be shown that the knowledge of the radial acceleration of the upper arm leads to estimating one coordinate of the elbow joint. This way, the redundant system is constrained in a way that it allows computation of the inverse kinematics of the arm. The necessary acceleration measurements have already been successfully implemented in the estimation of the angles of the lower extremities [4].

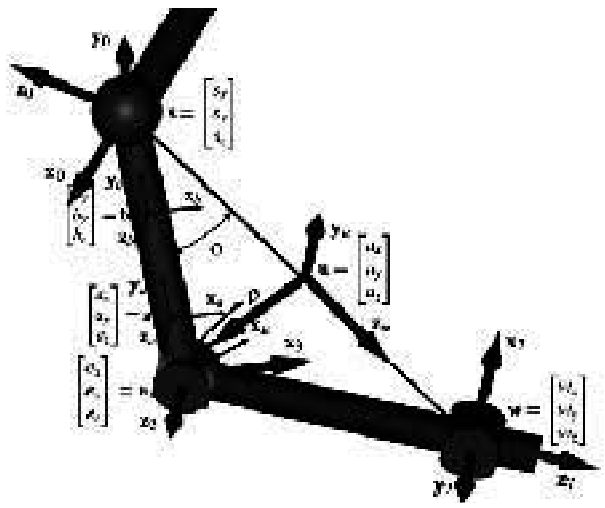

Fig. 2. Estimation of arm angles using hand pose data and upper arm radial acceleration

In Fig. 2, the abbreviations **s**, **e** and **w** define the shoulder position, elbow, and wrist joints with respect to the base coordinate frame.

Implying that the shoulder joint position is fixed and the base coordinate system is attached to the shoulder joint as indicated in the Fig. 1, the position of the elbow joint can simply be calculated as a function of shoulder angles and q_1 and q_2 upper arm length l_u as

$$\mathbf{e} = \begin{bmatrix} l_u \sin q_1 \cos q_2 \\ -l_u \cos q_1 \cos q_2 \\ -l_u \sin q_2 \end{bmatrix}. \qquad (1)$$

If the position of the shoulder joint is fixed, the radial acceleration at a point $\mathbf{a} = [\alpha_x \ \ \alpha_y \ \ \alpha_z]^T$ defined in the local frame of the upper arm, **E** (see Fig. 1 and Fig. 2), can be determined as

$$\begin{aligned}\ddot{a}_y = {} & g\cos q_1 \, cos q_2 + (l_u - \alpha_y)(\dot{q}_1^2 \cos q_2{}^2 + \dot{q}_2^2) + \\ & (a_x \sin q_3 + a_z \cos q_3)(\ddot{q}_1 \cos q_2 - 2\dot{q}_2 \dot{q}_3) + \\ & (a_x \cos q_3 - a_z \sin q_3)(\ddot{q}_2 + \dot{q}_1(2\dot{q}_3 - \dot{q}_1 \sin q_2)\cos q_2).\end{aligned} \qquad (2)$$

By considering that the attachment point of the accelerometer is as near as possible to the straight line connecting shoulder and elbow joints, the values of a_x and a_z become negligibly small. Under this condition, (2) can be re-written as

$$\ddot{a}_y \approx g\cos q_1 \, cos q_2 + (l_u - a_y)(\dot{q}_1^2 \cos q_2{}^2 + \dot{q}_2^2). \qquad (3)$$

The relationship in (3) shows that the radial acceleration at point $\mathbf{a} = [a_x \ a_y \ a_z]^T$ consists of two terms. The first term $g\cos q_1 \cos q_2$ is independent of the accelerometer position, whereas the second term, $(l_u - a_y)(\dot{q}_1^2 \cos q_2{}^2 + \dot{q}_2^2)$, depends on the distance between the local frame **E** attached on the upper arm at the elbow joint, and the accelerometer attachment point. At a point, where l_u equals a_y the second component vanishes. In this case, however, it is required that the accelerometer be positioned exactly over the shoulder joint, which is almost impossible. On the other hand, by adding a second accelerometer for measuring the radial acceleration of the upper arm in a second point at the location, $\mathbf{b} = [b_x \ b_y \ b_z]^T$, relative to the local coordinate frame **E** , we can obtain

$$\ddot{b}_y \approx g\cos q_1 \, cos q_2 + (l_u - b_y)(\dot{q}_1^2 \cos q_2{}^2 + \dot{q}_2^2). \qquad (4)$$

By performing simple algebraic manipulation, it is possible to estimate e_y coordinate of the elbow joint as

$$e_y = \frac{\ddot{b}_y (a_y - l_u) - \ddot{a}_y (b_y - l_u)}{g\,(b_y - a_y)} - l_u \cos q_1 \, cos q_2. \qquad (5)$$

As described below, e_y coordinate constrains the elbow position relative to a horizontal plane, and the measured hand pose is the sufficient condition for the computation of the arm inverse kinematics. In order to estimate the arm inverse kinematics, we will first analyze swivel conditions of the elbow joint about a circular arc as shown in Fig. 2. The normal vector of the swivel arc $\mathbf{z}_u$, which is parallel to the straight line between the shoulder and the wrist can be computed as

$$\mathbf{z}_u = \frac{\mathbf{w} - \mathbf{s}}{\|\mathbf{w} - \mathbf{s}\|}. \tag{6}$$

Based on simple trigonometry, we determine the radius of the swivel arc ρ, using the angle between the upper arm and the straight-line connecting the wrist and the shoulder joints

$$\phi = \arccos \frac{\|\mathbf{w} - \mathbf{s}\|^2 + l_u^2 - l_f^2}{2 l_u \|\mathbf{w} - \mathbf{s}\|} \tag{7}$$

as

$$\rho = l_u \sin \phi. \tag{8}$$

The center of the swivel arc defined in the base frame can then simply be determined as

$$\mathbf{u} = l_u \mathbf{z}_u \cos \phi. \tag{9}$$

Next, a unit vector $\mathbf{x}_u$, which is orthogonal to the normal vector of the swivel arc $\mathbf{z}_u$, and pointing from the straight-line connecting the wrist and the shoulder joints toward the elbow joint will be defined. The second component ${}^{(2)}\mathbf{x}_u$, of the vector $\mathbf{x}_u$ can be determined as

$$ {}^{(2)}\mathbf{x}_u = \frac{e_y - u_y}{\rho}. \tag{10}$$

Since we require vectors $\mathbf{x}_u$ and $\mathbf{z}_u$ to form an orthonormal basis, it is required that the following constraints be satisfied : $\|\mathbf{x}_u\| = 1$ and $\mathbf{x}_u \cdot \mathbf{z}_u = 0$. Using this conditions it is possible to determine the remaining components ${}^{(1)}\mathbf{x}_u$ and ${}^{(3)}\mathbf{x}_u$, of the $\mathbf{x}_u$ vector as

$$ {}^{(1)}\mathbf{x}_u = \frac{-u_x u_y {}^{(2)}\mathbf{x}_u + u_z \sqrt{\|\mathbf{u}\|^2 (1 - {}^{(2)}\mathbf{x}_u) - u_y^2}}{\|\mathbf{u}\|^2 - u_y^2}, \tag{11}$$

$$ {}^{(3)}\mathbf{x}_u = -\frac{u_x {}^{(1)}\mathbf{x}_u + u_y {}^{(2)}\mathbf{x}_u}{u_z}$$

The orthonormal vector $\mathbf{y}_u$ can simply be determined as a normalized cross product of vectors $\mathbf{z}_u$ and $\mathbf{x}_u$ as

$$\mathbf{y}_u = \frac{\mathbf{z}_u \times \mathbf{x}_u}{\|\mathbf{z}_u \times \mathbf{x}_u\|}. \tag{12}$$

The orthonormal vectors $\mathbf{x}_u$, $\mathbf{y}_u$ and $\mathbf{z}_u$ form a basis of a local coordinate frame $\mathbf{U}$ attached at the center of the swivel arc at a distance $\mathbf{u}$ from the shoulder joint. The position of the elbow joint expressed in the frame $\mathbf{U}$ is simply determined as $\mathbf{e}_U = [\rho \ 0 \ 0]^T$. Using the vectors that form the local basis, the transformation matrix between the local frame $\mathbf{U}$, and the base coordinate system $\mathbf{S}$, can be determined as

$$\mathbf{U} = \begin{bmatrix} \mathbf{x}_u & \mathbf{y}_u & \mathbf{z}_u & \mathbf{u} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{13}$$

Using the transformation matrix $\mathbf{U}$ and the elbow position expressed in the local frame, $\mathbf{e}_U$, the elbow position in the base coordinate frame $\mathbf{S}$ can be computed as

$$\mathbf{e} = \mathbf{U}\mathbf{e}_U. \tag{14}$$

Having computed the elbow position, it is now possible to determine the arm inverse kinematics. In order to compute the shoulder angles, a transformation matrix between the shoulder and the elbow coordinate frames $\mathbf{E}$ will be analyzed as follows:

$$\mathbf{E} = \begin{bmatrix} E_{11} & -s1c2 & E_{13} & l_u s1c2 \\ E_{21} & c1c2 & E_{23} & -l_u c1c2 \\ -c2c3 & s2 & c2s3 & -l_u s2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{15}$$

where $s1 = \sin q_1$, $s2 = \sin q_2$, $s3 = \sin q_3$, $c1 = \cos q_1$, $c2 = \cos q_2$ and $c3 = \cos q_3$. The expressions for some elements of matrix $\mathbf{E}$ were omitted due to their complexity and their insignificant role in computing the shoulder angles. On the other hand, matrix $\mathbf{E}$ can also be formed with the vectors forming the coordinate frame of the upper arm as

$$\mathbf{E} = \begin{bmatrix} \mathbf{x}_3 & \mathbf{y}_3 & \mathbf{z}_3 & \mathbf{e} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{16}$$

where

$$\mathbf{y}_3 = \frac{\mathbf{e} - \mathbf{s}}{\|\mathbf{e} - \mathbf{s}\|}, \quad \mathbf{z}_3 = \frac{(\mathbf{e} - \mathbf{s}) \times (\mathbf{w} - \mathbf{s})}{\|(\mathbf{e} - \mathbf{s}) \times (\mathbf{w} - \mathbf{s})\|}, \quad \mathbf{x}_3 = \frac{\mathbf{y}_3 \times \mathbf{z}_3}{\|\mathbf{y}_3 \times \mathbf{z}_3\|}. \tag{17}$$

By comparing equations (15) and (16) it is possible to estimate the shoulder angles as

$$q_1 = \arctan \frac{-{}^{(1)}\mathbf{y}_3}{{}^{(2)}\mathbf{y}_3}, \quad q_2 = \arcsin {}^{(3)}\mathbf{y}_3, \quad q_3 = \arctan \frac{{}^{(3)}\mathbf{z}_3}{{}^{(3)}\mathbf{x}_3}, \tag{18}$$

where arctan is the four quadrant inverse tangent function.

Since the elbow joint variable q_4 represents the only joint variable that affects the distance $\|\mathbf{w} - \mathbf{s}\|$, q_4 can be computed independently. We consider the normal vector of the plane containing the shoulder, elbow and wrist joints parallel to the elbow axis of rotation. Therefore, angle q_4 can be estimated trivially using the law of cosines as

$$q_4 = \arcsin \frac{l_u^2 + l_f^2 - \|\mathbf{w} - \mathbf{s}\|^2}{2 l_u l_f}. \tag{19}$$

At this point, only the wrist angles still remain to be estimated. Since the position and the orientation of the hand are measured directly, the transformation matrix $\mathbf{H}$ is known. Using the already estimated shoulder and elbow angles, the transformation matrix $\mathbf{T}_4$ is first computed. The wrist joint transformation matrix $\Omega = f(q_5, q_6, q_7)$ can be computed accordingly as

$$\Omega = \mathbf{T}_4^{-1}\mathbf{H}. \tag{20}$$

In order to compute the wrist angles, the matrix $\Omega = f(q_5, q_6, q_7)$ will be rewritten in an analytical form as

$$\Omega = \begin{bmatrix} \Omega_{11} & -\sin q_5 \cos q_6 & \Omega_{13} & 0 \\ \Omega_{21} & \cos q_5 \cos q_6 & \Omega_{23} & 0 \\ -\cos q_6 \sin q_7 & \sin q_6 & \cos q_6 \cos q_7 & l_f \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{21}$$

based on the relationships in Fig. 1. The expressions for some elements of matrix Ω were omitted due to their complexity and their insignificant role in computing the wrist angles. By observing the matrix Ω and considering (20), it is now possible to estimate wrist angles as

$$q_5 = \arctan \frac{-\Omega_{12}}{\Omega_{22}}, \quad q_6 = \arcsin \Omega_{32}, \quad q_7 = \arctan \frac{-\Omega_{31}}{\Omega_{33}}. \tag{22}$$

Ⅲ. RESULTS

The performance of the system utilizing derived calculation approach was evaluated on two intact volunteer subjects. Two 2-axis accelerometers ADXL202 (Analog Device, Inc.) were attached to the subject's upper arm. The accelerometer data proved to be highly noisy, therefore, a Kalman filter was additionally designed to estimate the angle values from the noisy measurements. Independently measured were the position and the orientation of the hand, using the contactless position acquisition system OPTOTRAK (Northern Digital, Inc.) with infrared markers that were placed on the wrist joint and on the first joint of the first and the little fingers. Two additional markers were attached to the shoulder and the elbow joints in order to obtain reference data necessary for the system validation. Fig. 3 shows the estimated and measured shoulder, elbow and wrist joint angles.

The noisy signals were the result of angle estimation without any prefiltering, the thick gray lines indicate estimated angles using the Kalman filter, while the thin black lines present the reference measurements. The accelerometer noise reflected in the shoulder angles measurements was relatively low, and could be significantly reduced using the Kalman filter. Since the accelerometer measurements were not used in the estimation of the elbow angle, q_4, the estimation of this angle was noise free. The noise was again reflected in the estimation of the wrist angles since the computation of matrix $\mathbf{T}_4$ requires the usage of the noisy measurements of shoulder angles. Nevertheless, with the implementation of the Kalman filter the sensor noise could be attenuated to satisfy the angle estimation requirements.

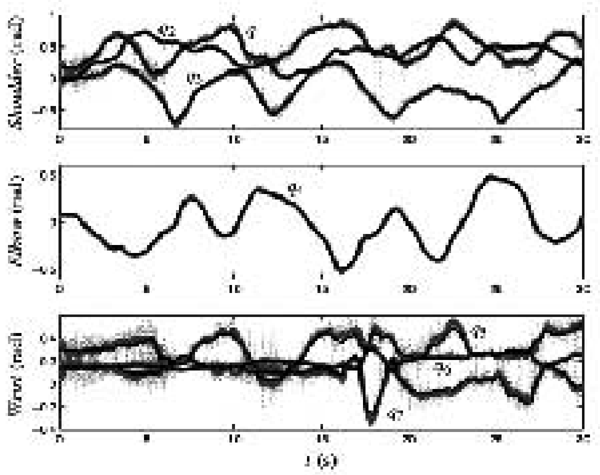

Fig. 3. Estimated and measured arm angles (without prefiltering-dotted line, Kalman filter output-thick gray line, reference-thin black.

Ⅳ. Summary and Conclusions

The paper depicts a novel technique for computation of the inverse kinematic model of the human arm. New approach is based on the measurements of the hand position and orientation, and radial acceleration of the upper arm. The algorithm gives sufficiently good estimates of the human arm joint angles, to be used in trajectory planning for rehabilitation purposes. It could be used in the evaluation of movement capabilities of patients with movement disorders and in control algorithms for artificial activation of upper extremity muscles using FES or exoskeleton devices.

The most limiting factor in the implementation of the inverse kinematics algorithm requires fixation of the shoulder joint. Assuming that the subject during upper extremity rehabilitation is usually seated, the fixation of the shoulder joint can simply be accomplished by using belts that are attached to the back of the seat.

The algorithm and the results presented in this paper prove the feasibility and computation of inverse kinematic model of the human arm based on hand pose and upper arm radial acceleration measurements. The algorithm is simple and numerically robust against the noise being present in the acquired accelerometer data.

Acknowledgment

The authors would like to thank Ministry of Education, Science and Sport of Republic of Slovenia for their financial support.

References

[1] J. Korein. *A geometric investigation of reach*, PhD Thesis, University of Pennsylvania, 1985.

[2] N. Tejima. Rehabilitation robotics: a review. *Adv. Robot*., 14: 551-564, 2000.

[3] D. Tolani, A. Goswami, and N. I. Badler. Real-time inverse kinematics techniques for anthropomorphic limbs. *Graph. Models*, 62: 353-388, 2000.

[4] A. Th. M. Willemsen, C. Frigo, and H. B. K. Boom. Lower extremity angle measurement with accelerometers-error and sensitivity analysis. IEEE Trans. Biomed. Eng., 38: 1186-1193, 1991.