Synthesis of standing-up trajectories using dynamic optimization

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Abstract

Dynamic optimization as a tool to compute standing-up trajectories was investigated. Sit-to-stand manoeuvres in five intact persons and five trans-femoral amputees were measured. Movements and ground reaction forces acting on the body were recorded. A five-segment 3D dynamic model of standing-up was developed. In each particular subject, the optimization criterion which yielded trajectories that best resemble the measured standing-up movement was determined. Since the intact persons used considerably different criteria in choosing the standing-up trajectories than the amputees, the optimal trajectories were computed by minimizing cost functionals (CF) with distinctive structures for each group of individuals. In intact persons, a unique cost functional was found which yielded realistic standing-up manoeuvres. In amputees, subject-specific sets of parameters indicating slightly different preferences in optimizing the effort of particular muscle groups were used.

Keywords: Standing-up; Dynamic optimization; Prostheses; Trans-femoral amputation

1. Introduction

Novel assistive devices are being developed which are intended to help persons with disabilities relearn or improve motor activities such as walking [1–3], standing-up [4], and reaching [5]. One approach to obtain the training trajectory for such assistive devices is to measure a physiotherapist’s induced motion. Such motion is not necessarily optimal in terms of consumed energy, minimization of peak muscle forces, or duration of movement. It is also not advisable to copy the motion of healthy persons. Persons with disabilities might not be able to reproduce the required speed or forces, since they do not have their senso-motoric capabilities completely preserved. Furthermore, the shape of a training trajectory depends on the anthropometric and musculoskeletal characteristics of a specific patient. A promising approach which could overcome some of the difficulties of training trajectory generation is dynamic motion optimization [6]. The present paper investigates the dynamic optimization of the standing-up motion. As standing-up from a sitting position requires considerable muscle effort [6] and can be performed using different strategies [7,8], control of standing-up assistive robots such as [4] is likely to benefit from dynamic optimization. The sit-to-stand process has been extensively described by other authors [9–11].

Several investigations have been performed to determine how human motion can be described within the frame of optimal control theory. Optimization algorithms mostly minimize cost functionals (CF) which consist of the time integral of the square norm of a quantity. Some quantities that were used in cost functionals are:

1. jerk (derivative of acceleration) of cartesian coordinates;
2. jerk of joint angles;
3. derivative of joint torques;
4. joint torques;
5. muscle forces and force derivatives.

Flash and Hogan [12] suggested that the arm reaching motion minimizes the time integral of the hand position jerk (CF1). Rosenbaum et al. [13] computed optimal movement in the joint space by replacing the position jerk in the cost functional [12] with the sum of joint angle jerks (CF2). Uno et al. [14] improved the optimization criterion by taking into account the arm dynamics (CF3). The minimum torque-change criterion presented in [14] minimized the sum of the joint torque derivatives. On the other hand,
the minimum-effort criterion used for robot motion planning in [15] might be more suitable for standing-up, as this activity requires considerably higher torques in the joints of the lower extremities compared to upper extremities (CF4). Hutchinson et al. [16] showed that the joint loads in the lower extremities due to segmental dynamics are considerably lower than the joint loads which arise from gravity. As the joint torques due to gravity are considerably lower in standing position than at the seat-off instant, it can be expected that the minimum-effort criterion would bring the body to a standing position as quickly as possible. It can be expected that a combination of minimum-effort and minimum torque-change cost functionals should successfully describe the standing-up manoeuvre.

Optimal rising from the sitting to the standing position was investigated by Pandy et al. [17] (CF5). In [17], optimal motion of a three segment 2D model was compared to measured standing-up of intact persons. Eight muscle groups were modeled and optimal neural excitation signals were computed by minimizing the muscle forces and their derivatives. Comprehensive models such as [17] aim for better understanding of the central nervous system coordination strategy. However, the trajectories of rehabilitation robots have to be subject-specific and the identification of musculoskeletal parameters is cumbersome and inaccurate. In contrast, only anthropometric parameters are needed for models that take into account solely the segmental dynamics. These parameters can be easily determined from a person’s weight and height [18].

It has not yet been investigated to what extent the optimization criterion depends on the type of disability. Dynamic motion optimization can be considered a suitable method for modeling of standing-up trajectories, if the variation of the cost functional in a particular group of individuals with disabilities is considerably smaller as compared to variation between groups with different types of disability and/or intact individuals. Intact individuals stand up symmetrically with respect to the sagittal plane [19]. Since prostheses are constructed for efficient walking [20], the prosthetic ankle and knee joints are almost completely passive during the sit-to-stand process. Thus, trans-femoral amputees stand up while leaning over their intact leg which bears most of the body weight. As the strategy of standing-up of intact persons differs from that in amputees, it can be expected that the optimization criteria used by these two groups are different.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Weight (kg)</th>
<th>Height (cm)</th>
<th>Age (year)</th>
<th>Years of prosthesis use</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>67</td>
<td>192</td>
<td>24</td>
<td>Intact</td>
</tr>
<tr>
<td>I2</td>
<td>77</td>
<td>172</td>
<td>24</td>
<td>Intact</td>
</tr>
<tr>
<td>I3</td>
<td>65</td>
<td>174</td>
<td>29</td>
<td>Intact</td>
</tr>
<tr>
<td>I4</td>
<td>77</td>
<td>182</td>
<td>26</td>
<td>Intact</td>
</tr>
<tr>
<td>I5</td>
<td>74</td>
<td>173</td>
<td>26</td>
<td>Intact</td>
</tr>
<tr>
<td>A1</td>
<td>90</td>
<td>172</td>
<td>55</td>
<td>18</td>
</tr>
<tr>
<td>A2</td>
<td>85</td>
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<td>A3</td>
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<td>7</td>
</tr>
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<td>A4</td>
<td>94</td>
<td>170</td>
<td>51</td>
<td>33</td>
</tr>
<tr>
<td>A5</td>
<td>77</td>
<td>177</td>
<td>39</td>
<td>32</td>
</tr>
</tbody>
</table>

All subjects were males.

were allowed to assume a comfortable foot placement. They were instructed to stand up at natural speed with their arms crossed over the chest. After three practice risings, ten risings were measured. The ground reaction forces acting on the body as well as the body motion were recorded (Fig. 1). The forces and torques under the feet were assessed using two AMTI force plates (AMTI, Inc., Newton, MA, USA). A JR3 six-axis robot wrist sensor (JR3, Inc., Woodland, CA, USA) mounted under the seat measured the force and torque reaction vectors. An Optotrak optical system (Optotrak, Northern Digital Inc., Waterloo, Canada) was used to acquire the body motion. Infrared markers were attached over the approximate centers of the ankle, knee, hip, L5/S1 intervertebral, shoulder, and elbow joints. Additionally, two markers were placed on each foot and two on the head.

A three-dimensional dynamic model was developed. The model consisted of five segments (Fig. 2): shanks, thighs and HAT (head, arms, trunk). Ankles and hips were modeled as three degrees of freedom (DOF) rotational joints while knees were represented with one DOF rotational joints. The recursive Newton–Euler formulation described in [21] was used to derive the model. In this manner, an 11 DOF open kinematic chain starting at the right ankle and finishing at the left ankle was modeled. The ground frame (x0, y0, z0) was placed in the right ankle. According to [21], 11 segment frames conforming to Denavit–Hartenberg convention were defined. The 11th segment frame was placed at the distal end of the kinematic chain. Ground reaction force and torque vectors were applied in the left ankle. The segmental anthropometric parameters were taken from [18], while the inertial parameters of prosthetic segments were measured. The average time-courses of the joint angles q_i (i = 1, ..., 11) were calculated from the trajectories of the ankle, knee, hip, and shoulder markers. The markers on the upper body were used to compute the inertial parameters of the HAT segment from the inertial parameters of the pelvis, trunk, lower arms, upper arms, and head segments [18].

A sequential quadratic programming optimization algorithm was chosen. NAG C numerical libraries were used (NAG LTD, Oxford, UK). The optimization program com-

2. Methods

Five intact persons and five persons with trans-femoral amputation of their left leg participated in the study (Table 1). The five amputees were well accustomed to the use of prostheses (average 23 years after amputation). The subjects sat on a commercially available bicycle seat whose height was adjusted to 90% of the distance from the ground to the approximate center of the knee joint. The subjects
computed the trajectories of the joint coordinates as well as the time-courses of the force and torque vectors acting in the left ankle. Since only the joint angles which place the left ankle of the model in the measured position represent a valid solution, a constraint function which immobilized the distal end of the kinematic chain was implemented. The trajectories obtained by the optimization program were parameterized by quintic B-splines [22]. The advantages of the trajectory approximation with B-splines in dynamic optimization are described in [15].

The cost functional $C$ was designed to minimize the integral of joint torques amplitudes (minimum-effort term $C_1$), the integral of torque changes (minimum torque-change term $C_2$), and the integral of difference between left and right ground reaction forces (term $C_3$):

$$C = \frac{1}{T} \int_{t_0}^{T_f} (C_1 + T^2 C_2 + C_3) \, dt,$$

$$C_1 = \tau^T E_L \tau + \lambda_L^T E_L \lambda_L, \quad C_2 = \dot{\lambda}_L^T D_L \dot{\lambda} + \lambda_L^T D_L \lambda_L,$$

$$C_3 = (\lambda_L - \lambda_R)^T S (\lambda_L - \lambda_R), \quad T = t_f - t_0 \quad (1)$$

In Eq. (1), vectors $\tau$, $\lambda_L$, and $\lambda_R$ have the following forms:

$$\tau = [\tau_1 \tau_2 \cdots \tau_{11}]^T,$$

$$\lambda_L = [f_{Lx} f_{Ly} f_{Lz} m_{Lx} m_{Ly} m_{Lz}]^T,$$

$$\lambda_R = [f_{Rx} f_{Ry} f_{Rz} m_{Rx} m_{Ry} m_{Rz}]^T \quad (2)$$

$\tau$ is the $11 \times 1$ vector of joint torques. The vectors $\lambda_L$ and $\lambda_R$ contain the three components of the force vector as well as of the torque vector in the left and right ankle, respectively. The components of $\lambda_L$ and $\lambda_R$ are given in ground frame coordinates. $E_L$, $D_L$, $D_R$, and $S$ are diagonal matrices. The initial time, $t_0$, of the optimization coincided with the instant of seat-off. The final time, $t_f$, was arbitrarily set to an instant when all joints were extended in standing configuration. The multiplication of the term $C_2$ by the square of the rising time, $T$, assured that the ratio between the terms of $C$ was not affected by the duration of standing-up.

The solution of the optimization problem was subject to the following constraints:

Fig. 1. Amputee AS during standing-up.
\[ H(q) \ddot{q} + h(q, \dot{q}) = \tau + J^T \lambda, \quad q_{\min} \leq q(t) \leq q_{\max}, \]
\[ q(t_0) = q_0, \quad \dot{q}(t_f) = \dot{q}_f, \quad \dot{q}(t_0) = \dot{q}_0, \]
\[ \dot{q}(t_f) = 0, \quad c(q(t)) = 0 \]  

(3)

where \( q \) is the 11 \times 1 vector of joint coordinates (Fig. 2); \( J \) the 6 \times 11 Jacobian matrix; \( H \) the inertial matrix; \( \dot{c} \) the vector of Coriolis and gravity terms; \( q_{\min} \) and \( q_{\max} \) are the lower and upper bound of joint coordinates, respectively; and \( c \) the constraint that immobilizes the position of the left ankle. The initial \( q(t_0) \) and final joint positions \( q(t_f) \) as well as initial joint velocities \( \dot{q}(t_0) \) were set to measured values. The final joint velocities \( \dot{q}(t_f) \) were set to zero.

During the standing-up process the joints that produce mainly motion in the sagittal plane have larger ranges of motion. We will call these joints sagittal in the rest of the paper. Sagittal joints were weighted separately, whereas to the other joints equal weights were assigned. In the intact subjects, the cost functional was designed to yield symmetry of movement. As symmetric movement requires low torques in the non-sagittal joints, only the minimum-effort term was non-zero in these joints.

\[ E_i(i) = E_0, \quad i = 2, 3, 6–9; \quad E_j(j) = E_0, \quad j = 4, 5 \]  

(4)
In Eq. (4), \( E_i(i) \) is the diagonal element of the matrix \( E_i \) weighting the torque in the \( i \)th joint. Similarly, \( E_j(j) \) is the \( j \)th diagonal element of the matrix \( E_j \). The constant \( E_0 \) was set to one. \( E_0 \) was a reference value for all the other elements of the weight matrices.

In the sagittal joints, the minimum-effort as well as the minimum-torque-change terms were used. The elements of \( D \) affecting the hip joint were set with respect to \( E_0 \), whereas the elements of \( D \) weighting the knee and ankle joints were defined with respect to the hip weights:

\[
\begin{align*}
D_i(5) &= D_i(10) = d_{\text{HP}} = D_{\text{HP}} E_0, \\
D_i(4) &= D_i(11) = d_{\text{KN}} = D_{\text{KN}} d_{\text{HP}}, \\
D_i(1) &= D_i(6) = d_{\text{AK}} = D_{\text{AK}} d_{\text{HP}}
\end{align*}
\]

where \( D_{\text{HP}}, D_{\text{KN}}, \) and \( D_{\text{AK}} \) are subject-specific values. The ratio between the minimum-effort and minimum torque-change weights for each particular sagittal joint were defined by the following subject-specific parameters:

\[
\begin{align*}
E_i(5) &= E_i(10) = e_{\text{HP}} = E_{\text{HP}} d_{\text{HP}}, \\
E_i(4) &= E_i(11) = e_{\text{KN}} = E_{\text{KN}} d_{\text{KN}}, \\
E_i(1) &= E_i(6) = e_{\text{AK}} = E_{\text{AK}} d_{\text{AK}}
\end{align*}
\]

where \( E_{\text{HP}}, E_{\text{KN}}, \) and \( E_{\text{AK}} \) are subject-specific parameters for hip, knee, and ankle joint, respectively.

In order to guarantee fully symmetric movement, the differences between the ground reaction forces in the left and right ankle joint were minimized in the vertical and horizontal direction:

\[
\begin{align*}
S(1) &= E_0, & S(2) &= -E_0
\end{align*}
\]

The weights of the cost functional for intact subjects that were not mentioned above were set to zero.

In the amputees, the optimization problem was designed quite differently, as the prosthetic knee and ankle are almost completely passive during standing-up. In the dynamic model, the knee torque and the torque vector in the left ankle were set to zero. Since the component of the ankle force in the direction of \( y_{11} \) replaced the effect of the left knee torque, it was determined by inverse dynamics. In this way, only the time-courses of the left ankle force components in the direction of \( x_{11} \) and \( z_{11} \) were computed by the optimization program. The size of the vector \( \lambda_L \) was reduced to \( 2 \times 1 \):

\[
\lambda_L = \begin{bmatrix} f_{11x} & f_{11z} \end{bmatrix}
\]

Since the standing-up of amputees is not symmetric with respect to the sagittal plane, the term \( C_3 \) was omitted from the cost functional \( C \). Instead, the amount of the body weight borne by the prosthesis was determined by adding the constraints:

\[
\lambda_L(t_0) = \lambda_{L0}, \quad \lambda_L(t_f) = \lambda_{LF}
\]

where \( \lambda_{L0} \) and \( \lambda_{LF} \) are the measured force components in the \( x_{11} \) and \( z_{11} \) directions, respectively. Due to asymmetric movement the torques in the non-sagittal joints are higher than in intact persons. Torque-change was minimized in these joints:

\[
D_i(i) = D_0, \quad i = 2, 3, 6-10
\]

The constant \( D_0 \) was set to one and replaced the role of \( E_0 \) in the intact persons. The weights for joints 1, 4, and 5 were defined similarly to Eqs. (5) and (6). Since the term \( C_3 \) was omitted, the control of the left ankle force trajectory was implemented in the \( D_0 \) and \( E_0 \) matrices:

\[
\begin{align*}
D_0(1) &= D_0(2) = d_{\text{GND}} = D_{\text{GND}} D_0, \\
E_0(1) &= E_{\text{GND}} d_{\text{GND}}, & E_0(2) &= 0
\end{align*}
\]

By varying the \( D_{\text{GND}} \) and \( E_{\text{GND}} \) subject-specific parameters, it was possible to determine the amplitude and time-course of the load on the prosthesis.

Since the mass and inertia of the HAT segment are considerably higher compared to the other segments, the position and orientation trajectories of the HAT segment (Fig. 2) were chosen to evaluate the quality of the computed optimal trajectories. An additional motivation was that the standing-up assistive robot [4] determines the HAT move-
Fig. 4. Measured (solid line) and computed (dashed line) standing-up of the amputee A5. The computed HAT is represented by the triangle; right hip—center of mass—left hip.

Fig. 5. Evaluation of computed standing-up in the intact subjects. Maximal and average errors in HAT position and HAT orientation in the sagittal (s), transverse (t), and frontal (f) planes are shown. The errors in the optimal HAT trajectories which were computed using the cost functional designed for intact persons (CF1) are compared to that computed using the cost functional designed for amputees (CF2).
3. Results and discussion

Optimal standing-up trajectories were calculated using different sets of weights in the cost functional. The calculated trajectories were compared to measured standing-up manoeuvres by stick figure animation (Figs. 3 and 4), joint angle, and joint torque trajectories. In the intact subjects, a set of weights was found which yielded adequately sim-ilar computed and measured movements (Table 2). In the amputees, it was not possible to determine a unique set of weights which would produce natural standing-up manoeuvres for all subjects (Table 2). It appears that the influence of the physical condition, training, and stump length on the sit-to-stand movement cannot be neglected. However, the difference between the corresponding cost functional weights in the amputees was significantly smaller than the difference between the weights in the amputees and in the intact subjects.

<table>
<thead>
<tr>
<th>Table 2: Cost functional parameters</th>
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<tbody>
<tr>
<td>Subject</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>Intact</td>
</tr>
<tr>
<td>A1</td>
</tr>
<tr>
<td>A2</td>
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<tr>
<td>A3</td>
</tr>
<tr>
<td>A4</td>
</tr>
<tr>
<td>A5</td>
</tr>
</tbody>
</table>

Fig. 6. Evaluation of computed standing-up in the amputees. Maximal and average errors in HAT position and HAT orientation in the sagittal (s), transverse (t), and frontal (f) planes are shown. The errors in the optimal HAT trajectories which were computed using the cost functional designed for amputees (CF2) are compared to that computed using the cost functional designed for intact persons (CF1).
In Figs. 5 and 6, the maximal and average differences between the measured and optimal HAT trajectories are shown for each intact subject and amputee. It was also investigated, how the cost functional designed for intact subjects (cost functional CF1) performs in the amputees and vice versa (Figs. 5 and 6). In intact subjects, all five sets of weights used in the cost functional for amputees performed similarly. The results of the cost functional with the weights of the amputee A2 (cost functional CF2) are shown in Fig. 5. The appropriate cost functionals performed significantly better than the cost functionals designed for the other group. The use of unsuitable cost functionals resulted in approximately two times higher errors in the intact subjects (Fig. 5) and three times higher errors in the amputees (Fig. 6). Since the use of specially designed cost functionals appears necessary, the rest of the paper will concentrate on the results obtained with the appropriate cost functionals.

The errors in the intact subjects were less than the errors in the amputees (Figs. 5 and 6). The maximal position errors were approximately 4 and 5 cm, whereas the maximal

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Fig. 7. Computed (solid line) trunk position and orientation in the intact person II. The mean measured trajectories (dashed line) and the lines denoting the plus/minus two standard deviations interval (fine dotted lines) are shown.
orientation errors were approximately 5 and 10° in the intact subjects and amputees, respectively.

In the amputees, the parameter $D_{GND}$ was set sufficiently high to reduce the influence of other weights on the ground reaction forces under the prosthesis. The parameter $E_{GND}$ was used to adjust the shape of the left ankle force time-course. The initial and final values of the left ankle force trajectory were fixed, thus by increasing $E_{GND}$ the trajectory curved towards zero. In both groups of subjects, the weights governing the lateral stability and body rotation around the vertical axis were assigned equal values ($E_0$ or $D_0$). $D_{HP}$ determined the values of sagittal joint weights with respect to the reference values $E_0$ or $D_0$. A sufficiently low $D_{HP}$ was selected in order to allow for experimenting with sagittal weights without compromising lateral stability. On the other hand, when $D_{HP}$ was set excessively low, it was not possible to control the sagittal motion. The motion in the sagittal plane was controlled by $E_{AK}$, $E_{KN}$, $E_{HP}$, $D_{AK}$ and $D_{KN}$. $E_{AK}$, $E_{KN}$, $E_{HP}$ determined how fast the torque curves of respective joints descended towards the final value. Thus, higher values of $E_{AK}$, $E_{KN}$, and $E_{HP}$ yielded faster motion of the respective joint. $D_{AK}$ and $D_{KN}$

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**Fig. 8.** Computed (solid line) trunk position and orientation in the amputee A5. The mean measured trajectories (dashed line) and the lines denoting the plus/minus two standard deviations interval (fine dotted lines) are shown.
reflected the priority between minimizing torques in the ankle, knee, and hip joints.

The amputees showed different preferences in choosing the trajectories of standing-up. Since the subject A3 had considerably less practice than the others (Table 2), his $E_{\text{GND}}$ was the highest. In the subject A3, the prosthesis bore 25% of the body weight during standing, while the average was 40%. High $D_{\text{KN}}$ values in the subjects A1 and A2 reflect considerate emphasis given to diminishing the effort of the knee muscles. A high $D_{\text{HP}}$ shows that the subject A4 preferred to optimize sagittal rising at the expense of lateral stability. Thus, he rose in 1.12 s, whereas the average duration of standing-up in amputees was 1.76 s. The subject A4 also used his prosthesis efficiently, since it bore 52% of the body weight during standing. Fig. 6 shows the highest HAT trajectory errors in the subject A1. The standing-up of the subject A1 was the slowest, as it took him 2.64 s. The fact that the subject A1 had more time to control the prosthesis movement showed in a fluctuating left ankle force which was difficult to model using dynamic optimization.

In Figs. 7 and 8, trajectories of the HAT position and orientation are presented. It can be observed that no excessively fast changes and no oscillations in the computed HAT trajectories are present. In the HAT position (coordinates $x$ and $y$) and orientation in the sagittal plane, low relative errors are present. Since in the other HAT trajectories smaller changes of amplitudes occur, some absolute errors appear as high relative error. The difference between the optimal and measured HAT trajectory is in general lower than the variability within an individual. Only the optimal trajectory of the orientation in the frontal plane leaves the two standard deviations confidence interval of the measured trajectory.

The results of the present study indicate that dynamic motion optimization can be used to compute the trajectories for standing-up assistive robots such as [4]. Taking into account solely the body dynamics and kinematics without modeling muscle behavior appears to yield sufficiently accurate trajectories. In this way, cumbersome identification procedures of patient’s musculoskeletal properties are not necessary.

Besides novel rehabilitation devices, other applications involving human motion might benefit from computing the trajectories of motion in advance instead of using measured data. Such applications are realistic computer graphics, ergonomics and humanoid robotics. It is easier to generate realistic human motion by specifying a cost functional than by designing each joint trajectory separately. Dynamic motion optimization might also be suitable in order to define an objective measure for progress of rehabilitation therapies.

A sensitivity analysis is not described in the present paper, as it would require considerable space due to the complexity of the dynamic model. Slight variations of computed trajectories are difficult to compare to measured trajectories, as the measurement of body motion are only accurate to a limited extent.

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References


